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THE MASSES AND PULSATIONS OF BL HERCULIS VARIABLES

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I. INTRODUCTION

The BL Herculis variables are primarily Population II stars found in galactic halo globular clusters, dwarf spheroidal galaxies, and the small Magellanic cloud. This class of variables pulsates in the radial fundamental mode at periods between 1 and 3 days with a few as long as 8 days. Their evolution places them on the above horizontal branch (AHB) or asymptotic giant branch (AGB) between the RR Lyrae and the W Virginis variables. There central He is exhausted and energy is supplied by both He and H burning shells (Gingold, 1976).

BL Herculis stars are of particular interest because they frequently exhibit light curve bumps on their rising or falling branch similar to those seen in the more massive, metal richer classical Cepheids. These bumps, observed to switch from descending to ascending light between $1^d.5$ and $1^d.7$, can be interpreted by the Simon and Schmidt (1976) hypothesis as a near resonance of the second overtone (Π_2) and the fundamental (Π_0) pulsation modes when the linear theory $\Pi_2/\Pi_0 \sim 0.5 \pm 0.03$. Based on nonlinear calculations for Cepheids by Stobie (1969a,b), this hypothesis predicts bumps before maximum light for $\Pi_2/\Pi_0 < 0.5$, and after maximum light for $\Pi_2/\Pi_0 > 0.50$.

A linear nonadiabatic pulsation study by King, Cox, and Hodson (KCH)(1981) for 0.55 and 0.75 $M_\odot$, King Ia composition ($X = 0.7$, $Z = 0.001$), is given on the Hertzsprung-Russell diagram shown in Figure 1. The dashed lines of constant Π_2/Π_0 show that in order for $\Pi_2/\Pi_0 = 0.5$ line to be near the observed bump phase transition period of $1^d.7$, the masses of BL Her variables must be less than 0.55 $M_\odot$. At masses greater than 0.55 $M_\odot$ lines of constant Π_2/Π_0 shift to periods longer than $1^d.7$. Also, because the line of constant $\Pi_2/\Pi_0 = 0.5$ at 0.55 $M_\odot$ is not a line of constant period, we would expect to see a much larger period range of $1^d.7$-$3^d.0$ for this bump phase transition than the observed range of $1^d.5$-$1^d.7$.

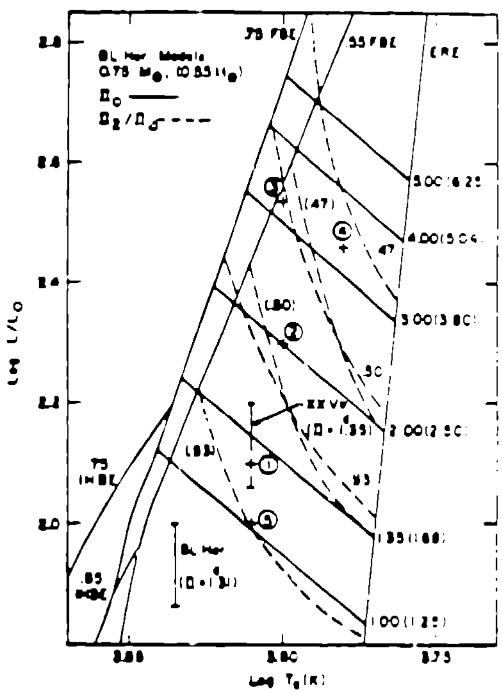

Fig. 1. On the HR diagram the fundamental (FBE) and overtone (1HBE) blue edges and an estimated red edge (FRE) are given for both $0.55 M_\odot$ and $0.75 M_\odot$. Lines of constant period (solid) and of constant period ratio (dashed) for both masses are indicated. Details for the two stars and the five nonlinear models (+ signs) are given by KCH.

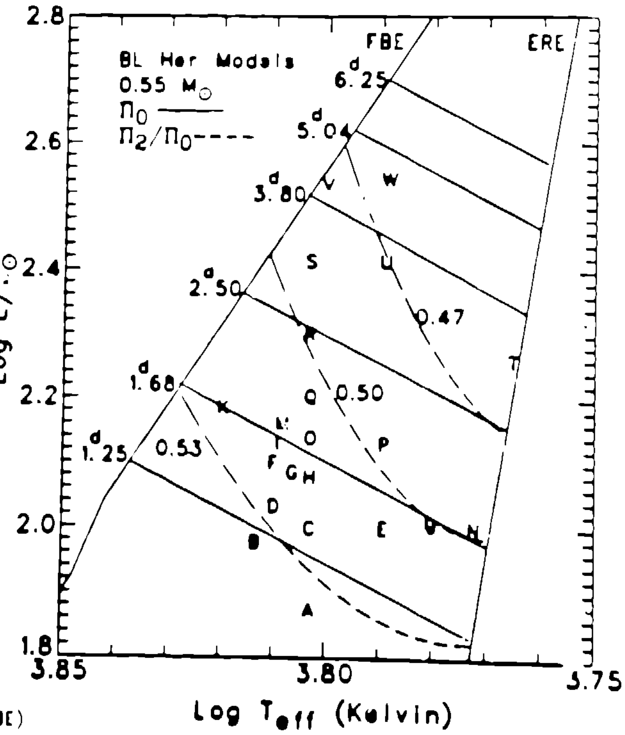

Fig. 2. Twenty-three nonlinear full amplitude fundamental mode models (cap. letters A thru W) are plotted with the $0.55 M_\odot$ linear results from Fig. 1. The alphabetic order indicates increasing length of nonlinear fundamental mode period

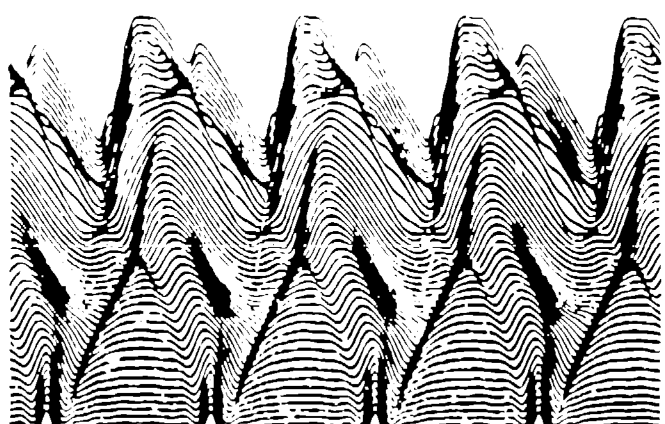

Fig. 3. Radial Velocity vs. Time for model F for four cycles of $\Pi_0 = 1^d.672$, with $\Pi_2/\Pi_0 = 0.521$
Velocity range (km/sec) core = −0.003 to 0.005
surface = −37 to 25

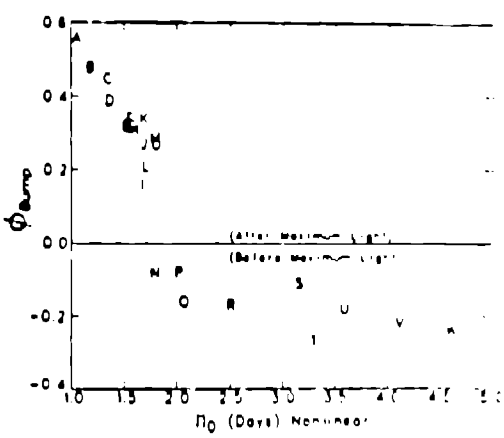

Fig. 4. Phase of the bump with respect to maximum light vs. the nonlinear fundamental mode period. The bump switches from after to before maximum light at $\sim 1^d.0$

II. NONLINEAR RESULTS

Theoretical light and velocity curves for 23 full amplitude, one space dimension models were constructed with the Population II composition King Ia as described by Hodson, Cox, and King (1982). These models are given by capital letters A through Z in Figure 2 together with the linear theory results for 0.55 $M_{\odot}$ given in Figure 1. The alphabetic order indicates increasing length of the nonlinear Π_0, and is preserved in subsequent figures.

Figure 3 gives the theoretical radial velocity over four fundamental mode periods for model F for each of the 50 Lagrangian mass zones in the star, with the scale progressively amplified for deeper layers. Figure 4 shows the bump phase versus nonlinear fundamental mode period obtained by measuring the phase before or after maximum light that the bump occurs on plots of light and velocity as shown in Figure 3. This nonlinear result gives the light curve bump transition $\sim 1^{d}.8$ in good agreement with observation.

III. THE FOURIER DECOMPOSITION

We obtained a better measure of the resonance phenomenon by considering the general shape of the surface theoretical light and velocity curves. Fourier decomposition into a principal frequency and n-1 harmonics given by

$$M_{bol},\ V_{rad} = A_0 + \sum_{i=1}^{n} A_i \cdot \cos(i\omega t - \Phi_i) \qquad (1)$$

was done similar to the study of Simon and Lee (1979). For the classical Cepheids, Simon and Lee (1979), and Simon, Lee, and Teays (1980), found sharp changes in $\Phi_{21} = \Phi_2 - 2\Phi_1$ and $A_{21} = A_1/A_2$ vs the observed period at 10^{d}. This period is also the bump phase transition period for these stars. For the BL Her stars, we plot Φ_{21} vs. Π_0 in Figure 5 for the radial velocity. The resonance exhibits itself as a sharp

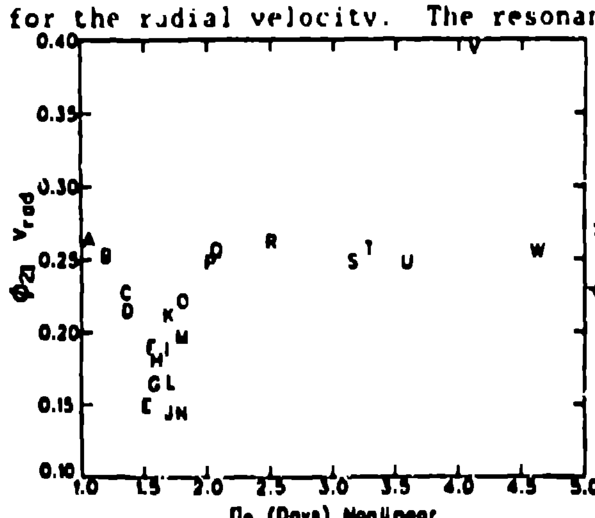

Fig. 5. Φ_{21} for surface radial velocity vs. Π_0. Note the sharp minimum at $\sim 1^{d}.7$

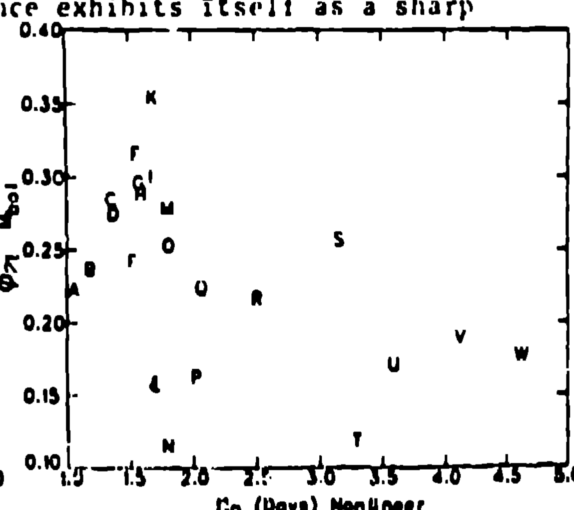

Fig. 6. Φ_{21} for M_{bol} vs. Π_0. Note the maximum at $\sim 1^{d}.7$.

minimum at $1^{d}.7$, whereas for the light curve Φ_{21} (Figure 6), we get a maximum at the resonance. The Φ_{21} and Π_0 correlation for light is not as good as for velocity, possibly because the luminosity may not always be well defined at every point in the cycle.

In Figure 7 the same Φ_{21} is plotted against the linear Π_2/Π_0. The resonance centers around $\Pi_2/\Pi_0 \cong 0.52$, not 0.50 as expected from the resonance hypothesis. However, a linear $\Pi_2/\Pi_0 = 0.52$ is consistent if the bump phase transition occurs at periods between $1^{d}.5$-$1^{d}.7$ (see Figure 2). If the nonlinear Π_2/Π_0, obtained from the periodic full amplitude solution, is used, then the resonance centers at $0.50 < \Pi_2/\Pi_0 < 0.51$ also shown in Figure 7. This is in closer agreement with the resonance hypothesis. This shift in the resonance occurs because the nonlinear Π_2/Π_0 is smaller, by as much as 0.012, than the linear counterpart for ratios from 0.49-0.52. In addition, the pulsationally stable Π_2 becomes substantially less stable in the full amplitude solution as the nonlinear $\Pi_2/\Pi_0 = 0.50$ is approached, as Simon (1977) suggested when a natural oscillator frequency (Π_2) becomes a forced oscillator driven by a harmonic (first harmonic) of Π_0.

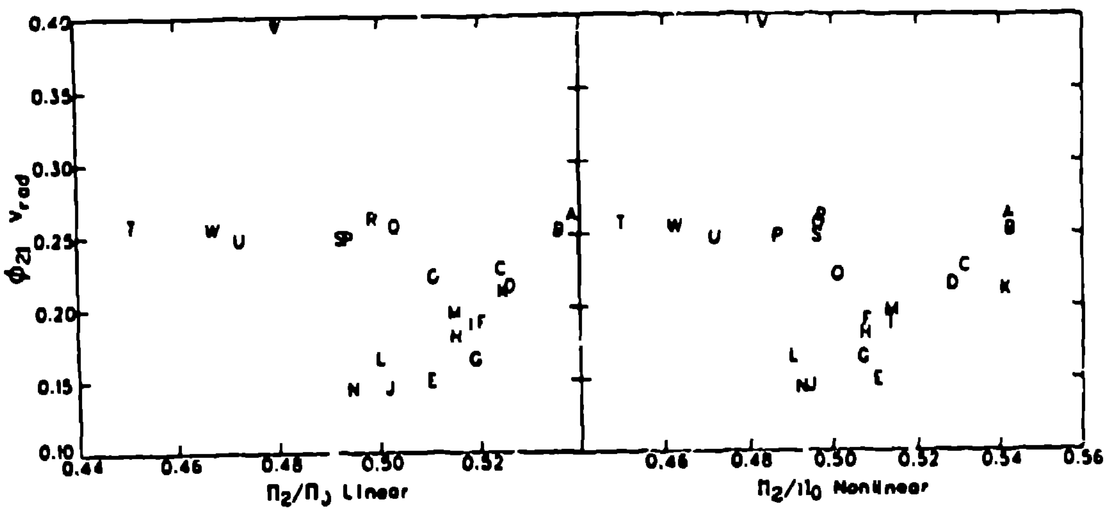

Fig. 7 Φ_{21} for the surface radial velocity vs. Π_2/Π_0 from linear and nonlinear theory, respectively. When the nonlinear period ratio is used, the resonance center shifts nearer to the expected center of Π_2/Π_0=0.50, if Π_2 is to couple harmonically with Π_0.

Another significant effect at the resonance center is a sharp increase in the amplitude ratio A_2/A_1 for the light at $1^{d}.7$ as shown in Figure 8. This increase is apparent up to the A_8/A_1 ratio and in the total light amplitude, indicating that the light curves become more skewed at higher amplitudes. For the radial velocity, A_2/A_1 decreases at $1^d.7$ as given in Figure 9. There is no significant correlation for higher ratios, however.

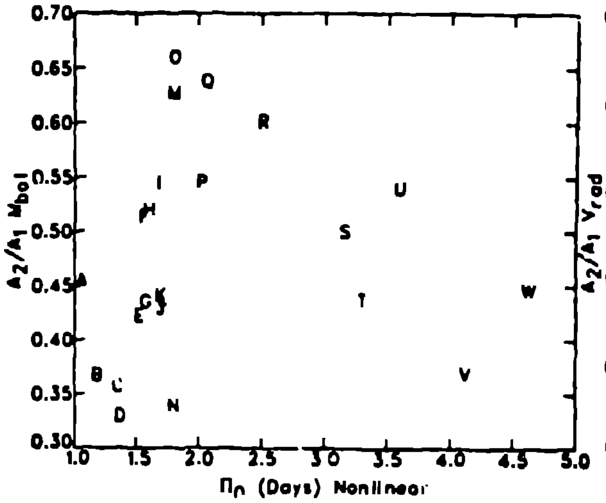

Fig. 8. A_2/A_1 for M_{bol} vs. Π_0. Note the large increase at $\sim 1^d.7$.

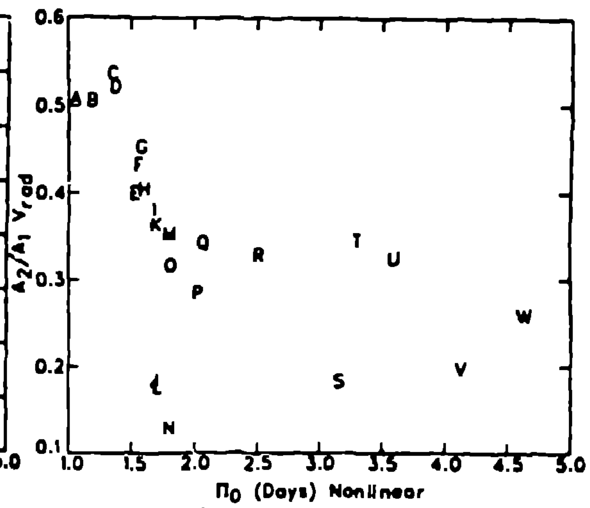

Fig. 9. A_2/A_1 for radial velocity vs. Π_0. Note the large decrease at $\sim 1^d.7$.

IV. CONCLUSIONS

From linear results, the masses of BL Her variables must be nearer to 0.55 $M_\odot$ than 0.75 $M_\odot$ if the bump phase transition (resonance) is to be located anywhere near the observed period range of $1^d.5-1^d.7$. The nonlinear results are consistent with the Simon resonance concept, but demonstrate that light and velocity curve shapes are a nonlinear phenomenon that require nonlinear period ratios to display the resonances only in the narrow, observed range of $1^d.5-1^d.7$. The mass near 0.55 $M_\odot$ is in good agreement with evolution calculations (Sweigart and Gross, 1976) and nonlinear pulsation studies of Carson, Stothers, and Vemury (1981) and Stothers (1981). Our recent efforts to Fourier analyze BL Her star observational data collected by Petersen have been unsuccessful due to poor phase coverage of available data.

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